

EXPLORING DATA
AND STATISTICS

5.8

Modeling with Quadratic Functions

GOAL 1 WRITING QUADRATIC FUNCTIONS

In Lesson 5.1 you learned how to graph a given quadratic function. In this lesson you will write quadratic functions when given information about their graphs.

EXAMPLE 1 Writing a Quadratic Function in Vertex Form

Write a quadratic function for the parabola shown.

SOLUTION

Because you are given the vertex $(h, k) = (2, -3)$, use the vertex form of the quadratic function.

$$y = a(x - h)^2 + k$$

$$y = a(x - 2)^2 - 3$$

Use the other given point, $(4, 1)$, to find a .

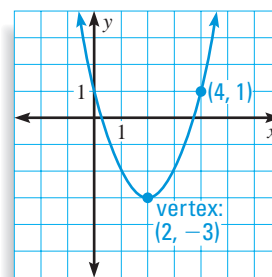
$$1 = a(4 - 2)^2 - 3 \quad \text{Substitute 4 for } x \text{ and 1 for } y.$$

$$1 = 4a - 3 \quad \text{Simplify coefficient of } a.$$

$$4 = 4a \quad \text{Add 3 to each side.}$$

$$1 = a \quad \text{Divide each side by 4.}$$

▶ A quadratic function for the parabola is $y = (x - 2)^2 - 3$.



EXAMPLE 2 Writing a Quadratic Function in Intercept Form

Write a quadratic function for the parabola shown.

SOLUTION

Because you are given the x -intercepts $p = -2$ and $q = 3$, use the intercept form of the quadratic function.

$$y = a(x - p)(x - q)$$

$$y = a(x + 2)(x - 3)$$

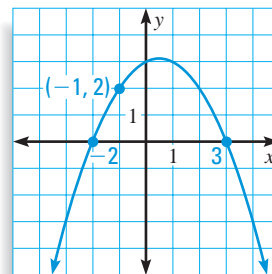
Use the other given point, $(-1, 2)$, to find a .

$$2 = a(-1 + 2)(-1 - 3) \quad \text{Substitute } -1 \text{ for } x \text{ and 2 for } y.$$

$$2 = -4a \quad \text{Simplify coefficient of } a.$$

$$-\frac{1}{2} = a \quad \text{Divide each side by } -4.$$

▶ A quadratic function for the parabola is $y = -\frac{1}{2}(x + 2)(x - 3)$.



What you should learn

GOAL 1 Write quadratic functions given characteristics of their graphs.

GOAL 2 Use technology to find quadratic models for data, such as the fuel economy data in Examples 3 and 4.

Why you should learn it

▼ To solve **real-life** problems, such as determining the effect of wind on a runner's performance in Ex. 36.



 **ACTIVITY**
Developing Concepts

Writing a Quadratic in Standard Form

In this activity you will write a quadratic function in standard form, $y = ax^2 + bx + c$, for the parabola in Example 2.

- 1 The parabola passes through $(-2, 0)$, $(-1, 2)$, and $(3, 0)$. Substitute the coordinates of each point into $y = ax^2 + bx + c$ to obtain three equations in a , b , and c . For instance, the equation for $(-2, 0)$ is:

$$0 = a(-2)^2 + b(-2) + c, \text{ or}$$

$$0 = 4a - 2b + c$$

- 2 Solve the system from **Step 1** to find a , b , and c . Use these values to write a quadratic function in standard form for the parabola.
- 3 As a check of your work, use multiplication to write the function $y = -\frac{1}{2}(x + 2)(x - 3)$ from Example 2 in standard form. Your answer should match the function you wrote in **Step 2**.


EXAMPLE 3 *Finding a Quadratic Model for a Data Set*

A study compared the speed x (in miles per hour) and the average fuel economy y (in miles per gallon) for cars. The results are shown in the table. Find a quadratic model in standard form for the data. ▶ Source: *Transportation Energy Data Book*

Speed, x	15	20	25	30	35	40
Fuel economy, y	22.3	25.5	27.5	29.0	28.8	30.0
Speed, x	45	50	55	60	65	70
Fuel economy, y	29.9	30.2	30.4	28.8	27.4	25.3

SOLUTION

Plot the data pairs (x, y) in a coordinate plane.

Draw the parabola you think best fits the data.

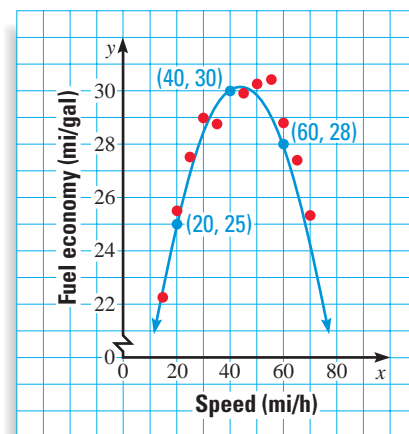
Estimate the coordinates of three points on the parabola, such as $(20, 25)$, $(40, 30)$, and $(60, 28)$.

Substitute the coordinates of the points into the model $y = ax^2 + bx + c$ to obtain a system of three linear equations.

$$400a + 20b + c = 25$$

$$1600a + 40b + c = 30$$

$$3600a + 60b + c = 28$$


STUDENT HELP
Look Back

For help with solving systems of three linear equations, see pp. 177, 217, and 231.

Solve the linear system. The solution is $a = -0.00875$, $b = 0.775$, and $c = 13$.

▶ A quadratic model for the data is $y = -0.00875x^2 + 0.775x + 13$.

**FOCUS ON
CAREERS**

**REAL LIFE
AUTOMOTIVE
DESIGNER**

Automotive designers help conceive of and develop new cars. They have to consider such factors as a car's appearance, performance, and fuel economy (the focus of Example 4).

CAREER LINK
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GOAL 2 USING TECHNOLOGY TO FIND QUADRATIC MODELS

In Chapter 2 you used a graphing calculator to perform linear regression on a data set in order to find a linear model for the data. A graphing calculator can also be used to perform *quadratic regression*. Quadratic regression produces a more accurate quadratic model than the procedure in Example 3 because it uses *all* the data points. The model given by quadratic regression is called the **best-fitting quadratic model**.

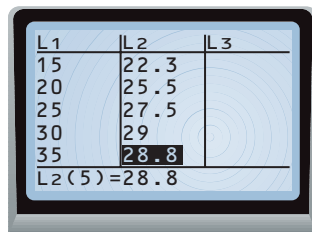
EXAMPLE 4 Using Quadratic Regression to Find a Model

FUEL ECONOMY Use the fuel economy data given in Example 3 to complete parts (a) and (b).

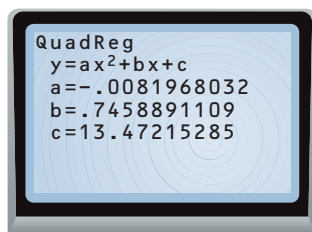
- Use a graphing calculator to find the best-fitting quadratic model for the data.
- Find the speed that maximizes a car's fuel economy.

SOLUTION

- Enter the data into two lists of a graphing calculator.



Use the quadratic regression feature to find the best-fitting quadratic model for the data.



▶ The best-fitting quadratic model is $y = -0.00820x^2 + 0.746x + 13.5$.

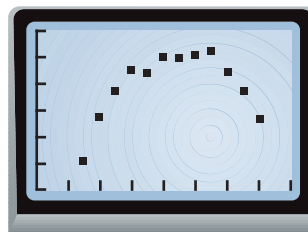
- You can find the speed that maximizes fuel economy by using the *Maximum* feature of a graphing calculator, as shown at the right.

You can also find the speed algebraically using the formula for the x -coordinate of a parabola's vertex from Lesson 5.1:

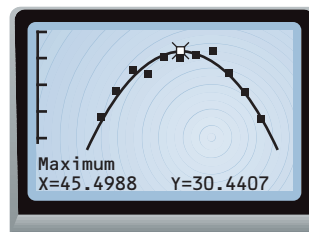
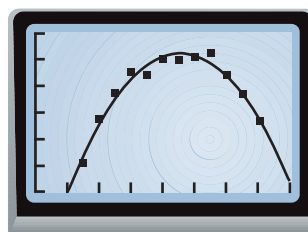
$$x = -\frac{b}{2a} = -\frac{0.746}{2(-0.00820)} \approx 45$$

▶ The speed that maximizes a car's fuel economy is about 45 miles per hour.

Make a scatter plot of the data. Note that the points show a parabolic trend.



Check how well the model fits the data by graphing the model and the data in the same viewing window.


STUDENT HELP
KEYSTROKE HELP

Visit our Web site
www.mcdougallittell.com
to see keystrokes for
several models of
calculators.

GUIDED PRACTICE

Vocabulary Check ✓

1. Complete this statement: When you perform quadratic regression on a set of data, the quadratic model you obtain is called the ?.

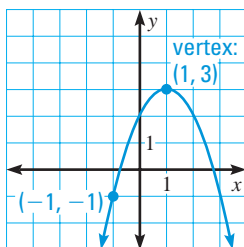
Concept Check ✓

2. How many points are needed to determine a parabola if one of the points is the vertex? if none of the points is the vertex?

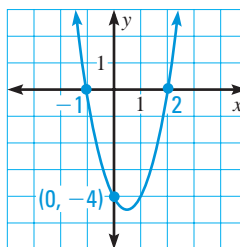
Skill Check ✓

Write a quadratic function in the specified form for the parabola shown.

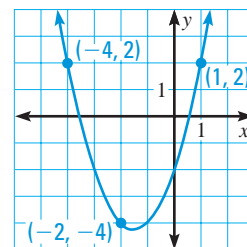
3. vertex form




4. intercept form



5. standard form



6.  **REAL ESTATE** The table shows the average sale price p of a house in Suffolk County, Massachusetts, for various years t since 1988. Use a system of equations to write a quadratic model for the data. Check your model by performing quadratic regression on a graphing calculator.

Years since 1988, t	0	2	4	6	8	10
Average sale price (thousands of dollars), p	165	154.5	124.5	115	128	165



DATA UPDATE of *Boston Globe* data at www.mcdougallittell.com

PRACTICE AND APPLICATIONS

STUDENT HELP

Extra Practice

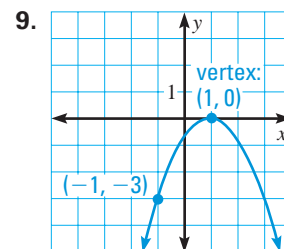
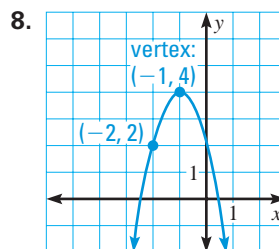
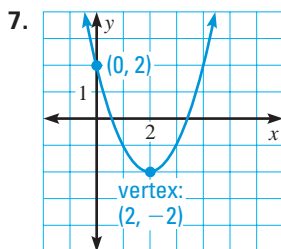
to help you master skills is on p. 947.

STUDENT HELP

HOMEWORK HELP

- Example 1:** Exs. 7–15, 34
Example 2: Exs. 16–24, 35
Example 3: Exs. 25–33, 36–38
Example 4: Exs. 37, 38

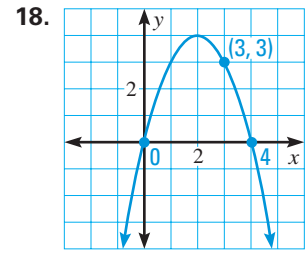
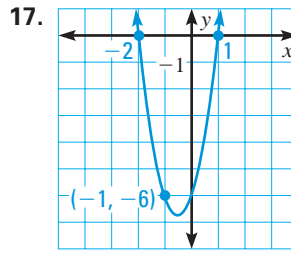
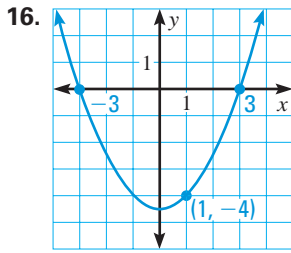
WRITING THE VERTEX FORM Write a quadratic function in vertex form for the parabola shown.



WRITING THE VERTEX FORM Write a quadratic function in vertex form whose graph has the given vertex and passes through the given point.

10. vertex: $(2, -1)$
point: $(4, 3)$
11. vertex: $(-4, 6)$
point: $(-1, 9)$
12. vertex: $(4, 5)$
point: $(8, -3)$
13. vertex: $(0, 0)$
point: $(-2, -12)$
14. vertex: $(1, -10)$
point: $(-3, 54)$
15. vertex: $(-6, -7)$
point: $(0, -61)$

WRITING THE INTERCEPT FORM Write a quadratic function in intercept form for the parabola shown.



WRITING THE INTERCEPT FORM Write a quadratic function in intercept form whose graph has the given x -intercepts and passes through the given point.

19. x -intercepts: 1, 4
point: (3, 2)

20. x -intercepts: -2 , 2
point: $(-4, 8)$

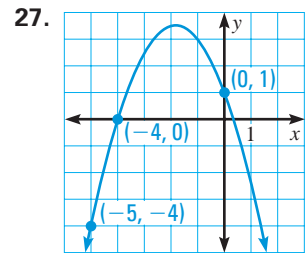
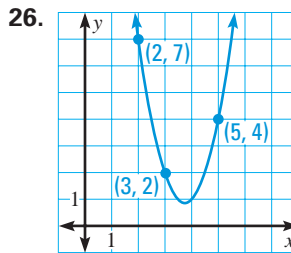
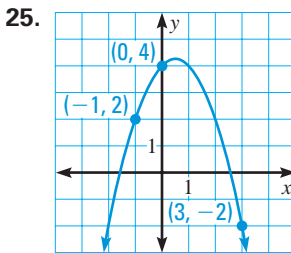
21. x -intercepts: -1 , 6
point: $(1, -20)$

22. x -intercepts: -10 , -8
point: $(-7, -15)$

23. x -intercepts: 3, 9
point: $(14, 77)$

24. x -intercepts: -5 , 0
point: $(-3, 18)$

WRITING THE STANDARD FORM Write a quadratic function in standard form for the parabola shown.



WRITING THE STANDARD FORM Write a quadratic function in standard form whose graph passes through the given points.

28. $(-1, 5)$, $(0, 3)$, $(3, 9)$

29. $(1, 2)$, $(3, 4)$, $(6, -8)$

30. $(-2, -1)$, $(1, 11)$, $(2, 27)$

31. $(-4, -7)$, $(-3, 3)$, $(3, -21)$

32. $(-3, -4)$, $(-1, 0)$, $(9, -10)$

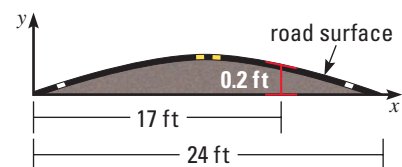
33. $(-6, 46)$, $(2, 14)$, $(4, 56)$

34. **BOTANY** *Amaranth* is a type of vegetable commonly grown in Asia, West Africa, and the Caribbean. When amaranth plants are grown in rows, the height that the plants attain is a quadratic function of the spacing between plants within a row. According to one study, the minimum height of the plants, about 16 cm, occurred when the plants were spaced about 27 cm apart. The study also found that the plants grew to about 20 cm when spaced about 40 cm apart. Write a quadratic model giving the plant height h as a function of the spacing s .

► Source: Center for New Crops and Plant Products, Perdue University

35. **TRANSPORTATION** The surfaces of some roads are shaped like parabolas to allow rain to run off to either side. (This is also true of football fields; see Exercise 52 on page 254.) Write a quadratic model for the surface of the road shown.

► Source: Massachusetts Highway Department



Not drawn to scale

STUDENT HELP

HOMEWORK HELP
Visit our Web site
www.mcdougallittell.com
for help with problem
solving in Exs. 34 and 35.

FOCUS ON PEOPLE



REAL LIFE **MARK MCGWIRE** hit 70 home runs during the 1998 Major League Baseball season, breaking Roger Maris's record of 61. McGwire's longest home run traveled 545 ft (166 m).

APPLICATION LINK
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Test Preparation

- 36. RUNNING** The table shows how wind affects a runner's performance in the 200 meter dash. Positive wind speeds correspond to tailwinds, and negative wind speeds correspond to headwinds. Positive changes in finishing time mean worsened performance, and negative changes mean improved performance. Use a system of equations to write a quadratic model for the change t in finishing time as a function of the wind speed s . ▶ Source: *The Physics of Sports*

Wind speed (m/sec), s	-6	-4	-2	0	2	4	6
Change in finishing time (sec), t	2.28	1.42	0.67	0	-0.57	-1.05	-1.42

- 37. AGRICULTURE** Researchers compared protein intake to average shoulder and kidney weight for a group of pigs. The results are shown in the table. Use systems of equations to write quadratic models for the shoulder weight s and kidney weight k as a function of the protein intake p . Check your models using the quadratic regression feature of a graphing calculator.

▶ Source: *Livestock Research for Rural Development*

Protein intake (g/day), p	195	238	297	341	401	427
Shoulder weight (g), s	8130	8740	9680	9690	9810	8990
Kidney weight (g), k	239	287	288	334	379	373

- 38. BASEBALL** The table shows the distance (in meters) traveled by a baseball hit at various angles and with different types of spin. (In each case the initial speed of the ball off the bat is assumed to be 40 m/sec.) Use systems of equations to write three quadratic models—one for each type of spin—that give the distance d as a function of the angle A . Check your models using the quadratic regression feature of a graphing calculator. ▶ Source: *The Physics of Sports*

Angle	10°	15°	30°	36°	42°	45°	48°	54°	60°
Distance (backspin)	61.2	83.0	130.4	139.4	143.2	142.7	140.7	132.8	119.7
Distance (no spin)	58.3	79.7	126.9	136.6	140.6	140.9	139.3	132.5	120.5
Distance (topspin)	56.1	76.3	122.8	133.2	138.3	139.0	137.8	132.1	120.9

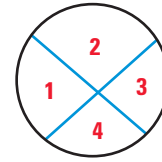
- 39. MULTI-STEP PROBLEM** The table shows the time t it takes to boil a potato whose smallest diameter (that is, whose shortest distance through the center) is d . ▶ Source: Dr. Peter Barham, University of Bristol

Diameter (mm), d	20	25	30	35	40	45	50
Boiling time (min), t	27	42	61	83	109	138	170

- Find the ratios $\frac{t}{d}$. Does boiling time vary directly with diameter? Explain.
- Find the ratios $\frac{t}{d^2}$. What do you notice?
- Use the result of part (b) to write a quadratic model for t as a function of d . Find the time needed to boil a potato whose smallest diameter is 55 mm.

★ Challenge

40. **GEOMETRY CONNECTION** Let R be the maximum number of regions into which a circle can be divided using n chords. For example, the diagram shows that $R = 4$ when $n = 2$. Copy and complete the table. Then write a quadratic model giving R as a function of n .



n	0	1	2	3	4	5	6
R	?	?	4	?	?	?	?

EXTRA CHALLENGE

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MIXED REVIEW

EVALUATING EXPRESSIONS Evaluate the expression for the given value of the variable. (Review 1.2 for 6.1)

41. $x^2 - 4$ when $x = 3$ 42. x^5 when $x = 2$
 43. $3u^3 + 10$ when $u = -4$ 44. $-v^4 + 2v + 7$ when $v = -1$

SOLVING SYSTEMS Solve the system using either the substitution method or the linear combination method. (Review 3.2)

45. $x - y = 4$ 46. $2x - y = 0$ 47. $3x + 2y = -2$
 $x + y = 2$ $5x + 3y = 11$ $4x + 7y = 19$

48. **HEALTH** You belong to a health maintenance organization (HMO). Each year, you pay the HMO an insurance premium of \$1800. In addition, you pay \$15 for each visit to your doctor's office and \$10 for each prescription. Write an equation for the annual cost C of your health plan as a function of your number v of office visits and number p of prescriptions. (Review 3.5)

QUIZ 3*Self-Test for Lessons 5.7 and 5.8*

Graph the inequality. (Lesson 5.7)

1. $y > x^2 + 2$ 2. $y \geq -x^2 - x + 3$ 3. $y \leq 2x^2 - 12x + 15$

Graph the system of inequalities. (Lesson 5.7)

4. $y \geq x^2$ 5. $y > x^2 - 2x - 3$ 6. $y > -x^2 - 4x$
 $y \leq 2x^2 - 1$ $y < -x^2 + 2x + 3$ $y \leq x^2 + 7x + 10$

Write a quadratic function in the specified form whose graph has the given characteristics. (Lesson 5.8)

7. vertex form
vertex: $(5, -2)$
point on graph: $(4, 0)$
8. intercept form
 x -intercepts: $-3, 1$
point on graph: $(2, -5)$
9. standard form
points on graph:
 $(-4, 8), (-2, 1), (2, 5)$
10. **COMPUTERS** Using an algorithm called *insertion sort*, a common minicomputer can sort N numbers from least to greatest in t milliseconds where $t = 0.00339N^2 + 0.00143N - 5.95$. How many numbers can the minicomputer sort in less than 1 second (1000 milliseconds)? Write your answer as an inequality. (Lesson 5.7)